Hydrino like states in graphene and Aharonov-Bohm field

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We study the dynamics of fermions on graphene in presence of Coulomb impurities and Aharonov-Bohm field. Special emphasis is given to the formation of hydrino like states and its lifting of degeneracy due to the presence of AB field. The flux of the AB field can be tuned to make the low angular momentum hydrino states stable against decay. The zero limit physics of the two coupling constants α_G and Φ involved in the system is discussed.

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Graphene, a 2-dimensional form of allotrope, made up of single layer of carbon atoms, has been a platform to study the various exotic features [1] of fermions which is usually not seen in most condensed matter systems. The exotic behavior is attributed to the unique lattice structure of the allotrope, which can be understood from the fact that the different layers of graphite are so loosely bound that it slides one above another. The bonding between the atoms of the same layer are strong enough to form a stable structure. It can be seen when struck by a pencil lead on a paper. The mark on the paper can be considered as a thin layer of carbon atoms. The recent experimental formation of graphene in 2004 [2] has made it possible to test several features in this system. For example, in semiconductor or in other material, the energy-momentum relation for the electrons with effective mass m^* are given by $E = p^2/2m^*$. But for monolayer graphene without any external perturbation or distortion, the dispersion relation for the electrons near a Dirac point are given by $E = \pm c_G|\boldsymbol{p}|$ with velocity c_G. This linear dispersion relation was a clue that the charge carriers in graphene could be described by massless Dirac equation. However, mass term for the fermions could arise if somehow the two sub lattices of the graphene are made distinguishable.

One of the exotic behaviors is that, the mobility of charge carriers in graphene is approximately independent of the carrier density. The constancy of the mobility means that the conductivity in graphene is proportional to the carrier density. The list of unusual phenomena in graphene is even extended in quantum Hall effect, which can not be explained by non-relativistic quantum mechanics. The integer quantum Hall effect occur at half integral values of the filling factor, which is therefore termed as anomalous [1, 3]. This effect is common in massless monolayer graphene and massive bilayer graphene. The Landau levels which are important to describe quantum Hall effect are also different from what is seen in non-relativistic quantum mechanics. In constant magnetic field B background perpendicular to the plane, the electron motion in metal or semiconductor is quantized as $E = \hbar\omega^*(n + 1/2), n = 0, 1, 2, ...$ The holes also have same behavior but with negative energy. Here $\hbar$ is the reduced Plank constant and $\omega^* = eB/m^*$ is the cyclotron frequency. But graphene has completely different Landau levels [3]. In monolayer graphene, it is given by $E_G = \mathrm{sgn}(n)c_G\sqrt{2e\hbar|n|B}$. The occurrence of the $E_G = 0$ states are the consequence of chirality [4] of the Dirac fermions. The degeneracy of the $E_G = 0$ state is however same as the $E_G \neq 0$ state. On the other hand, for bilayer graphene the spectrum is of the form $E_G = \mathrm{sgn}(n)\hbar\omega_G\sqrt{|n|(|n|+1)}, n = 0, \pm1, \pm2, ...$ The $E_G = 0$ levels are still present but this time the degeneracy is double than that of $E_G \neq 0$ levels.

All the above features are indicative of the fact that the dynamics of charge carriers in graphene is described by Dirac fermions. The tight binding model [5] of graphene indeed show that the dynamics of electron and hole are dictated by (2+1)-dimensional Dirac fermion in the continuum limit of the lattice spacing. Graphene is thus considered to be a condense matter analog of QED (see the introduction of [6]). However, it has lot of differences with the standard QED. Depending upon the situation, fermions could have mass or zero rest mass as mentioned before. The velocity of the zero rest mass fermions are $c_G \approx c/300$, which is much less than the light velocity c in vacuum. Due to this small Fermi velocity c_G, the fine structure constant is much higher $\alpha_G \approx 300\alpha$. Obviously the perturbation scheme of QED seems to be not applicable to graphene system due such a high coupling constant α_G.

In this work, we report another possibility of exotic phenomena of massive fermions on graphene, which is the formation of hydrino like states. But due to the high coupling constant α_G, the so called hydrino states may have complex eigenvalues for low angular momentum values, which will make it dissipative. We therefore consider a Aharonov-Bohm (AB) field, which will rescue the low angular momentum states and will form stable bound states. Hydrino state is the tightly bound states of Hydrogen atom with binding energy $E_B > 13.6eV$, where normal ground state energy for Hydrogen atom is $E_G = -13.6eV$. Experimental study has also been reported recently in support of this hydrino states (see [7] and its refs.). Theoretical model of hydrino states can be found by solving Klein-Gordon equation in 3 space

dimensions. Recently it has been shown that, in $(2+1)$-dimensions, Dirac equation may give rise to hydrino like states [8]. Besides the hydrino states, the large fine structure constant α_G of graphene also may make the normal bound state eigenvalue complex. Our inclusion of a AB field will also make the normal bound states stable.

The existence of deeply bound states can be explained in terms of a consistent mathematical formulation known as self-adjoint extensions [9]. In this method the commonly used Dirichlet boundary condition at origin of the wave-function, that the wave-function is zero at origin, is generically extended to a less restricted boundary condition. Usually the wave-function at the origin is considered to have a certain power law behavior without violating the square-integrability condition. This larger domain then allows other states to immerse out as a viable solutions.

We start our discussion with the fermions on graphene in presence of a gauge potential A_μ, described by a two component Dirac equation

$$(i\partial\!\!\!/ + A\!\!\!/ - M)\,\psi(t;r,\phi) = 0 \tag{1}$$

We set the unit in which Fermi velocity $c_G \approx 10^6 m/s = 1$ and $\hbar = 1$. Here $\partial\!\!\!/ = \gamma^\mu\partial_\mu$, $A\!\!\!/ = \gamma^\mu A_\mu$ and we take the representation of the gamma matrices in terms of Pauli matrices as $\gamma^0 = \sigma^3$, $\gamma^1 = i\sigma^1$ and $\gamma^2 = i\sigma^2$. Since we are considering Coulomb impurities and Aharonov-Bohm field, our gauge potential A_μ is the Coulomb potential $A_0 = \alpha_G/r$ and AB field $A_\phi = -\Phi/r$. The mass M of the fermion could generate due to the external perturbation, which makes the two sub-lattices of the honeycomb lattice structure of graphene distinguishable or by other means [10]. The the hopping of the electrons between two sub-lattices makes the energy difference nonzero. In order to solve (1) we need to know the condition under which the radial hamiltonian extracted from (1) is symmetric. This condition can be easily found to be

$$\lim_{r\to 0} r\phi_1^\dagger(r) i\sigma^2\phi_2(r) = 0\,, \tag{2}$$

where ϕ_1 and ϕ_2 are two component radial spinors. To construct a domain for our problem we need to construct it in such a way that it respect the constraint (2) and at the same time it becomes equal to its adjoint domain. We mentioned in the introduction that, the mathematical formulation which serve this purpose is known as self-adjoint extensions and it has been recently discussed in case of graphene with Coulomb impurities in [11] and a generalized boundary condition for the bound state solution has been obtained.

We do not go into detail analysis of it, rather we give here an intuitive argument to show that the boundary condition can be parameterized by $U(1)$ group. The short distance behavior of the two independent radial wave-functions $\psi_1(r)$ and $\psi_2(r)$ has the form

$$\begin{aligned}\lim_{r\to 0}\psi_1(r) &\sim \mathcal{A} r^{-1/2+\xi}\\ \lim_{r\to 0}\psi_2(r) &\sim \mathcal{B} r^{-1/2-\xi}\,,\end{aligned} \tag{3}$$

where $\mathcal{A}$ and $\mathcal{B}$ are the two component constants, $\xi = \sqrt{(m+\Phi)^2 - \alpha_G^2}$, and $m = \pm 1/2, \pm 3/2, ...$ For $\Phi = 0$, (3) is reduced to the short distance behavior of radial function of [8]. Note that both the solutions are square-integrable at $r = 0$ if

$$0 \le 2|Re(\xi)| < 1 \tag{4}$$

Technically, the point $r \to 0$ is called quantum mechanically incomplete [9] and the potential corresponding to the radial Hamiltonian is called in limit circle case. On the other hand only one solution is square-integrable near $r \to \infty$. So the potential is in limit point case at infinity. This means, there can have 1-parameter family of self-adjoint extensions. Since both solutions are square-integrable at $r \to 0$, then any linear combinations of the two will also be square-integrable. We take a generalized boundary condition of the form

$$\lim_{r\to 0}\psi(r) \equiv \mathcal{C}_1(\theta) r^{-1/2+\xi} + \mathcal{C}_2(\theta) r^{-1/2-\xi}\,, \tag{5}$$

where the constants, $\mathcal{C}_1(\theta)$ and $\mathcal{C}_2(\theta)$, which are now dependent on the free parameter θ of the $U(1)$ group, be such that (5) satisfies the constraint (2). Then θ is called the self-adjoint extension parameter. The normal bound state solutions can be obtained by setting the coefficient of the singular term of (5) to zero, i.e., $\mathcal{C}_2(\theta) = 0$. This gives the bound state solution of the form

$$E_{n,m} = M\mathrm{sgn}(\alpha_G)/\sqrt{1 + \alpha_G^2/(n + \sqrt{(m+\Phi)^2 - \alpha_G^2})^2}. \tag{6}$$

Note that for $\Phi = 0$ the above solution reduces to the result [8, 13]. Now comes the exotic solutions of the problem, which can be obtained by setting the 1st term of (5) to zero but keeping the singular term. In this case the solutions are square-integrable and the bound state eigen-value is given by

$$E_{n,m} = M\mathrm{sgn}(\alpha_G)/\sqrt{1 + \alpha_G^2/(n - \sqrt{(m+\Phi)^2 - \alpha_G^2})^2}\,, \tag{7}$$

which reduces to the result of [8] for $\Phi = 0$. This is the so called hydrino like states. Note that in presence of the magnetic flux, the angular momentum degeneracy is lifted. As is evident the states with spectrum (7) are tightly bound than the states with eigenvalues (6). we have two parameters in our system, one is fine structure constant α_G and other one is the flux Φ, whose limiting behavior can be investigated. We see that the limit $\Phi \to 0^\pm$ is smooth and from (6) and (7) we get

$$\lim_{\Phi\to 0^+} E_{n,m} = \lim_{\Phi\to 0^-} E_{n,m} = \lim_{\Phi=0} E_{n,m} \tag{8}$$

But the similar limit for α_G shows that

$$\lim_{\alpha_G \to 0^+} E_{n,m} \neq \lim_{\alpha_G \to 0^-} E_{n,m} \neq \lim_{\alpha_G = 0} E_{n,m} \qquad (9)$$

Note that the first inequality of (9) can be obtained from (6) or (7), but to get the second inequality we need to compare our result with [12].

To summarize, we considered the quantum dynamics of fermion on graphene in presence of Coulomb impurities and AB field background. Possible generalized boundary condition has been established based on intuited arguments. The fermions are trapped in the combined potential. We showed that the deepest bound state solutions, known as hydrino like states, are nothing but a consequence of a specific choice of boundary condition. It can be rigorously shown by self-adjoint extensions. The limit $\Phi \to 0$ is shown to be smooth but the limit of the fine structure constant is not smooth so can not be used to get the dynamics of the fermions in only AB field background. Due to large value of the fine structure constant α_G of graphene, the low angular momentum states $E_{n,m}$ may become complex when only Coulomb field is present. But the presence of the AB field may rescue it and make the states stable.

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